HIGHWALL STABILITY ANALYSIS ON A PC[1]

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Abstract. Conventional methods of slope stability analysis were found to be inadequate for analyzing highwalls consisting of different strata. A new method was developed to calculate the factor of safety for dry highwalls and the model was used to develop computer software for a personal computer. Presently, the program analyzes plane and circluar failures. Unlike the conventional random search technique, the critical slip surface is determined by a pattern optimation technique.

Additional Key Words: Highwall stability, Surface mining

Introduction

Analysis of abandoned highwalls is important because of the dangers a failure of the highwall poses to life and property. The conventional approach is to use available limiting equilibrium analysis of Bishop (1955), Janbu (1954), or Morgenstern and Price (1965) developed for slope stability problems. The above methods require that the shape and location of the potential slip surface be specified; or the location is determined by different optimization methods depending on the program algorithms. In the case of highwalls consisting of a number of strata, the above methods are not only inefficient but also, sometimes they do not yield the actual slip surface. In this paper, in order to overcome this problem, the factor of safety is derived by considering available shear strength and mobilized shear stress along an arbitrary failure surface. However, for practical applications, the solution algorithm still needs to assume the shape of failure surface. Plane and circular failures have been fully analyzed in this paper using the new concept. The critical failure surface, a sliding surface which has the minimum factor of safety, is then obtained adusting the surface in order to minimize the factor of safety. Minimization of the factor of safety is achieved by an optimization technique which is a mathematical process of maximizing or minimizing a function. The new development to the problem of highwall stability analysis has been implemented on a PC.

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Theoretical basis

Referring to Figure 1, $y_1 = f(x)$ and $y_2 = g(x)$ are the equations of the slip surface and slope surface respectively. The toe of the slope is taken as the origin of the X-Y coordinate system. For an infinitely small slice of thickness dx, $d\tau_a$ is the available shear strength and $d\tau_m$ is the shear stress mobilized along the failure surface y_1. The force equilibrium equation for the system can be written as:

$$d\tau_m - d\tau_a = 0 \tag{1}$$

From Figure 1, the following relationships for $d\tau_m$ and $d\tau_a$ can be established:

$$d\tau_m = \gamma(y_2 - y_1)\sin\alpha \; dx \tag{2}$$

$$d\tau_a = c \; d\ell + \tan\phi \; \{\gamma(y_2 - y_1) \; \cos\alpha \; dx\} \tag{3}$$

where, c is cohesion, ϕ is angle of internal friction ($\mu = \tan\phi$) and γ is unit weight of the stratum; ℓ is the arc length along y_1. Substituting Equations (2) and (3) into Equation (1), we get:

$$\gamma(y_2 - y_1)\sin\alpha \; dx - c \; d\ell - \mu(\gamma(y_2 - y_1)\cos\alpha \; dx\} = 0 \tag{4}$$

In addition, the following relationships can be derived from differential calculus:

$$\cos\alpha = \frac{dx}{d\ell} \tag{5}$$

$$\sin\alpha = y_1' \frac{dx}{d\ell} \tag{6}$$

$$d\ell = \sqrt{1 + y_1'^2} \; dx \tag{7}$$

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Factor of safety

In order to quantify the margin of safety relative to a state of imminent failure, the strength parameters c and μ are to be reduced by a factor F known as factor of safety. Hence,

$$c = \frac{c_m}{F} \text{ and } \mu = \frac{\mu_m}{F} \tag{8}$$

where, c_m and μ_m are mobilized cohesion and coefficient of friction respectively (we will use the symbol c and μ to denote c_m and μ_m throughout the paper). Using Equations (5), (6), (7) and (8), Equation (4) can be written as:

$$\frac{\gamma(y_2 - y_1)y_1'}{\sqrt{1 + y_1'^2}} dx - \frac{c}{F}\sqrt{1 + y_1'^2}\, dx - \frac{\mu}{F}\frac{\gamma(y_2 - y_1)}{\sqrt{1 + y_1'^2}} dx = 0 \tag{9}$$

Equilibrium of the mass bound by OEG (Figure 1) is:

$$\int_{x_o}^{x_n} \{F\gamma(y_2 - y_1)y_1' - c(1 + y'^2) - \mu\gamma(y_2 - y_1)\}dx = 0 \tag{10}$$

Factor of safety F can be obtained from Equation (10) as:

$$F = \frac{\int_{x_o}^{x_n} \{c(1 \times y_1'^2) + \gamma\mu(y_2 - y_1)\}\, dx}{\int_{x_o}^{x_n} \gamma(y_2 - y_1)y_1'\, dx} \tag{11}$$

Factor of safety for layered highwall. The above formulation of the safety factor is for the single layered highwall. In practice, however, highwalls consist of different strata (Figure 2). A factor of safety for an n layered highwall can be formulated as:

$$F = \frac{\sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} \{c_i(1+y_1'^2) + \gamma\mu_i(y_2-y_1)\}\, dx}{\int_{x_o}^{x_n} \gamma(y_2 - y_1)y_1'\, dx} \tag{12}$$

where γ is taken to be the weighted average unit weight of all the strata in the highwall. This is done to avoid complexity in evaluating the above integrals. Note: c_i and μ_i are for the layers in which the failure plane is passing.

Plane shear failure

In the case of plane shear failure of a rectilinear highwall of height H and slope angle θ (Figure 3), the following relationships for y_2 and y_1 can be derived:

$$y_2 = x \tan\theta \quad \text{for} \quad x \le \frac{H}{\tan\theta} \tag{13}$$

$$y_2 = H \quad \text{for} \quad x \ge \frac{H}{\tan\theta} \tag{14}$$

$$y_1 = x \tan\beta \tag{15}$$

The result of integrating each term of Equation (12) by using Equations (13), (14) and (15) for y_2 and y_1, is:

$$A = \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} \{c_i(1+y_1'^2)dx$$

$$= \sec^2\beta \sum c_i\ (x_i - x_{i-1}) \tag{16}$$

$$B = \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} \gamma\mu_i(y_2 - y_1)dx$$

$$= \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} \gamma\mu_i y_2 dx - \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} \gamma\mu_i y_1 dx \tag{17}$$

or

$$B = \gamma\sum \mu_i[\frac{\tan\theta}{H}(x_i^2 - x_{i-1}^2) - \frac{\tan\beta}{2}(x_i^2 - x_{i-1}^2)]$$

$$x \le \frac{H}{\tan\theta} \tag{18}$$

$$B = \gamma\sum \mu_i[H(x_i - x_{i-1}) - \frac{\tan\beta}{H}(x_i^2 - x_{i-1}^2)]$$

$$x \ge \frac{H}{\tan\theta} \tag{19}$$

$$C = \int_{x_o}^{x_n} \gamma(y_2 - y_1)y_1' dx$$

$$= \gamma\int_{o}^{H/\tan\theta} y_2 y_1' dx + \gamma\int_{H/\tan\theta}^{x_n} y_2 y_1 dx - \int_{x}^{x_n} y_1 y_1' dx \tag{20}$$

or

$$C = \gamma[\frac{\tan\theta\tan\beta}{2}\left(\frac{H}{\tan\theta}\right)^2 + H\tan\beta\left(x_n - \frac{H}{\tan\theta}\right) - \tan^2\beta\,\frac{x_n^2}{2}] \tag{21}$$

In order to establish the critical sliding surface, the factor of safety F = (A + B)/C is to be minimized for the unknown variable x_n. This is done by an optimization method described later.

Circular failure

For a circular failure surface of radius r having the center at the coordinates x_c y_c (Figure 4), the sliding surface is:

$$y_1 = y_c - \sqrt{r^2 - (x - x_c)^2} \tag{22}$$

The result of integrating each term of Equation (12) by using Equations (13), (14) and (22) for y_2 and y_1, is:

$$A = \Sigma \frac{c_i r}{2} \left\{ \log\left[\frac{r + (x_i - x_c)}{r - (x_i - x_c)}\right] - \log\left[\frac{r + (x_{i-1} - x_c)}{r - (x_{i-1} - x_c)}\right] \right\} \quad (23)$$

$$B = B_1 - B_2 \quad (24)$$

for $x \leq \frac{H}{\tan\theta}$

$$B_1 = \gamma \Sigma \mu_i [\tan\theta \frac{x_i^2}{2} - y_c x_i + \frac{x_i - x_c}{2} \sqrt{r^2 - (x_i - x_c)^2} + \frac{r^2}{2} \sin^{-1} \frac{x_i - x_c}{r}] \quad (25)$$

$$B_2 = \gamma \Sigma \mu_i [\tan\theta \frac{x_{i-1}^2}{2} - y_c x_{i-1} + \frac{x_{i-1} - x_c}{2} \sqrt{r^2 - (x_{i-1} - x_c)^2} + \frac{r^2}{2} \sin^{-1} \frac{x_{i-1} - x_c}{r}] \quad (26)$$

and for $x \geq \frac{H}{\tan\theta}$

$$B_1 = \gamma \Sigma \mu_i [H x_i - y_c x_i + \frac{x_i - x_c}{2} \sqrt{r^2 - (x_i - x_c)^2} + \frac{r^2}{2} \sin^{-1} \frac{x_i - x_c}{r}] \quad (27)$$

$$B_2 = \gamma \Sigma \mu_i [H x_{i-1} - y_c x_{i-1} + \frac{x_{i-1} - x_c}{2} \sqrt{r^2 - (x_{i-1} - x_c)^2} + \frac{r^2}{2} \sin^{-1} \frac{x_{i-1} - x_c}{r}] \quad (28)$$

$$C = C_1 + C_2 + C_3 - C_4 \quad (29)$$

$$C_1 = \gamma \left\{ \tan\theta [\left(\frac{r^2}{2} + x_c^2\right) \sin^{-1} \frac{x - x_c}{r} - \frac{1}{2}(x + 3x_c) \sqrt{r^2 - (x - x_c)^2}] \right\}_0^{H/\tan\theta} \quad (30)$$

$$C_2 = \gamma \left\{ - x_c \tan\theta [x_c \sin^{-1} \frac{x - x_c}{r} - \sqrt{r^2 - (x - x_c)^2}] \right\}_0^{H/\tan\theta} \quad (31)$$

$$C_3 = \gamma \left\{ - H[\sqrt{r^2 - (x - x_c)^2}] \right\}_{H/\tan\theta}^{x_n} \quad (32)$$

$$C_4 = \gamma \left\{ x_c x - \frac{1}{2} x^2 - y_c \sqrt{r^2 - (x - x_c)^2} \right\}_0^{x_n} \quad (33)$$

From the above equations, the factor of safety can be determined as $F = (A + B)/C$. The four unknowns, x_c, y_c, r and x_n, in the objective function, F, are reduced to two unknowns by the following transformations (Figure 4):

$$r = \frac{H^2 + x_n^2}{2(x_n \cos\omega - H \sin\omega)} \quad (34)$$

$$x_c = x_n - r\cos\omega \quad (35)$$

$$y_c = H + r\sin\omega \quad (36)$$

where, ω is the angle between the radius EF of the arc and slope line EG.

By using the optimization method, x_n and ω are determined for the minimum value of F.

Optimization technique

For optimizing the function F, a pattern search method has been used in the program (Hooke and Jeeves 1967). This ia a straightforward method and intuitively appealing. Unlike the classical gradient method of optimization, the pattern search technique does not require finding the derivative of the function. Because of this, there is no assumption of continuity or the existence of first and second derivatives of the function in a specified interval.

Unlike the random search technique, the pattern search method employs sequential examination of the trial solutions. Each trial solution is compared with the "best" obtained up to that point and the next trail solution is determined by a strategy. The method evaluates the objective function at each base point. The procedure of going from a given base to the next base is called a move. A move is a success if the objective function decreases; otherwise, it is a failure. The pattern seach routine involves two types of moves. The first is an exploratory move designed to acquire knowledge about the behavior of the function. In the second move, the pattern move, the actual minimization of the function is performed by utilizing the result obtained from the exploratory move. Figures 5 and 6 show the detailed flow charts of the pattern and exploratory moves respectively. Table 1 explains the variables used in the flow charts.

Program structure

The program is written in TURBO BASIC and consists of input, factor of safety, optimization, graph and print subroutines. The input subroutine requests necessary data, such as, H, θ, γ, c_i, μ_i and h_i (thickness of each stratum from the bottom of the highwall). The input data are checked for errors and an option is provided to store them on the disk. A separate subroutine calculates F for plane and circular failure with initial values of x_n = (H/tanθ) + 1 for plane failure and x_n = (H/tanθ), ω = 0 for circular failure. The optimization subroutine optimizes F by repeatedly calling the factor of safety routine. The minimum F is the current F which is less than the previous F for a minimum step size (δ) 0.01. A graph subroutine displays the slope with the failure surface on the screen. The print subroutine prints the output on the screen as well as on the line printer. Table 2 shows a demonstration run of the program. For plane failure, the minimum factor of safety is found to be 0.87 and the intersection of the slope surface and sliding surface (x_n) is located at a distance of 61.7 feet from the origin. For circular failure, the radius of the slip circle is 87.8 feet having its center at -42, 77. The minimum factor of safety for the circular failure is 0.72 and x_n is at a distance of 45.5 feet from the origin.

For computing the factor of safety, the following steps are performed.

For plane failure:

1. Enter initial value of $x_n = (H/\tan\theta) + 1$.

2. Calculate $\tan\beta = H/x_n$.

3. Calculate x_i (i = 1 to n - 1) from the equation:

$$x_i = \frac{\Sigma h_i}{\tan\beta}$$

4. Calculate A, B, and C from Equations (16), (18) or (19), and (21).

5. Calculate the factor of safety $F = (A + B)/C$.

For circular failure:

1. Enter initial values of $x_n = H/\tan\theta$ and $\omega = 0$.

2. Calculate r, x_c, and y_c using Equations (34), (35), and (36).

3. Calculate x_i(i = 1 to n - 1) from equation:

$$x_i = x_c + \sqrt{r^2 - (\Sigma h_i - y_c)^2} \qquad (38)$$

4. Calculate A, B, and C from Equations (23), (24), and (29).

5. Calculate the factor of safety $F = (A + B)/C$.

The factor of safety subroutine consists of steps 2 through 5. The optimization subroutine changes values of x_n and ω and calls the factor of safety subroutine to calculate F. The current F is compared with the previous F for undertaking further moves. The operation will stop when the current F is less than the previous F. Figures 5 and 6 explain the operation of the optimization subroutine.

Conclusion

Analysis of highwall stability by using the conventional force equilibrium method has been successfully accomplished on a PC. The random search techniques to locate the critical slip surface have been replaced by an optimization method. Computational procedures and applications have proved that it is possible to run a comprehensive analysis of highwall stability on a PC involving circular or plane failures.

The approach described here to calculate factor of safety is very powerful. Though only simple cases like plane and circular failures have been fully analyzed herein, other modes of failures, such as, plane failure with tension cracks (no water in the tension crack), non-circular slip surfaces or even wedge failures can be analyzed by using the same procedure. It needs further modification and development of the current method if noncircular slip surfaces or wedge failures are to be incorporated.

The method described in this paper is one of the limit equilibrium methods and that only force equilibrium is being satisfied. The method does not account for moment equilibrium. So, the method does not satisfy complete equilibrium condition. The method has not been yet tested in the field.

References

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Table 1. Explanations of the variables used in flow charts.

The variables Ψ, ϑ and φ are points in n dimensional space.

ϑ	The previous base point.
Ψ	The current base point.
φ	The base point resulting from the current move.
$F(\Psi)$	The functional value at the base point.
$F(\varphi)$	The functional value for this move.
F	The functional value befor this move.
Δ	The current step size.
δ	Minimum step size.
ρ	Reduction for step size.
φ	One of the coordinate values.
$E:F,\varphi$	A program E(exploratory routine) will change F and φ.

Table 2. Demonstration run of the program.

Geometrical Information of the Highwall	
Height of the highwall (ft.):	70
Slope angle of the highwall:	70
Strata Information	
Strata 1.	
Thickness (ft.)	25
Cohesion (lb/ft^2)	800
Friction angle	25
Unit weight (lb/ft^3)	160
Strata 2.	
Thickness (ft.)	15
Cohesion (lb/ft^2)	700
Friction angle	20
Unit weight (lb/ft^3)	170
Strata 3.	
Thickness (ft.)	30
Cohesion (lb/ft^2)	900
Friction angle	25
Unit weight (lb/ft^3)	180
Plane Failure	
x(n) (ft.)	61.7
Factor of Safety	0.87
Circular Failure	
Radius of slip circle (ft.)	87.8
x_c	-42
y_c	77
x(n) (ft.)	45.5
Factor of safety	0.72

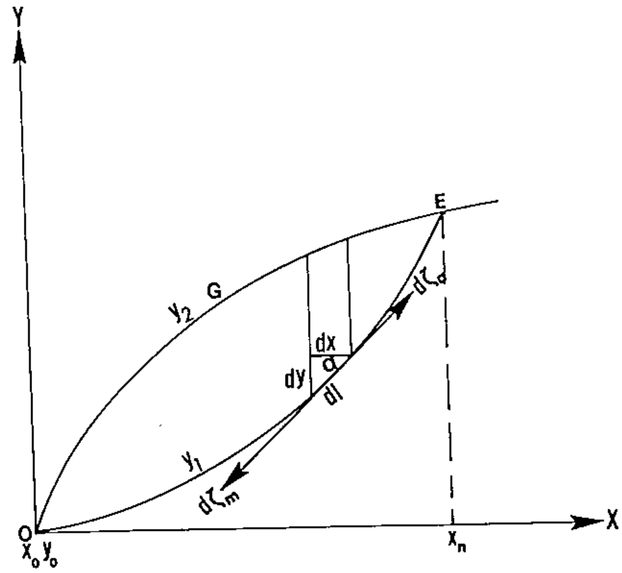


Figure 1. Basic conventions and definitions.

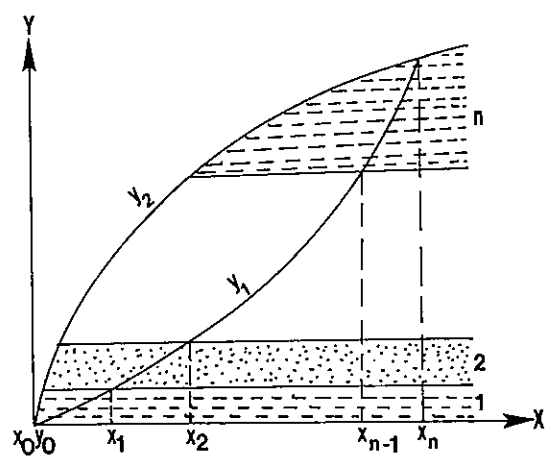


Figure 2. Conventions for layered highwall.

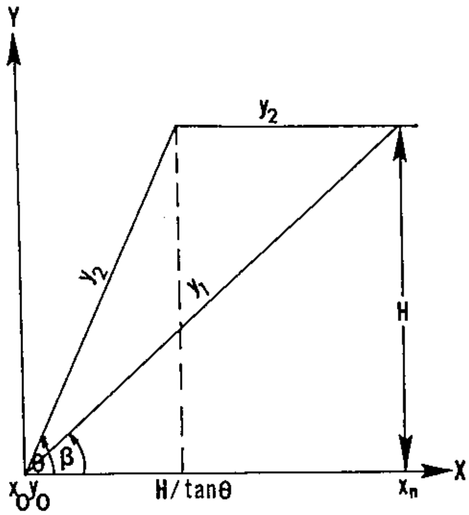


Figure 3. Plane failure.

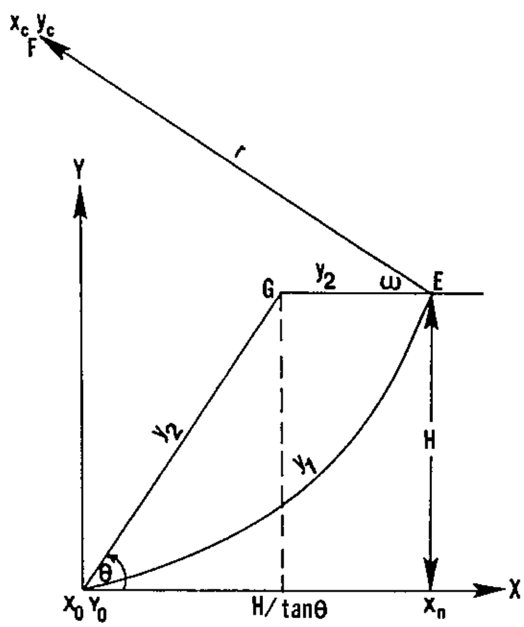


Figure 4. Circular failure.

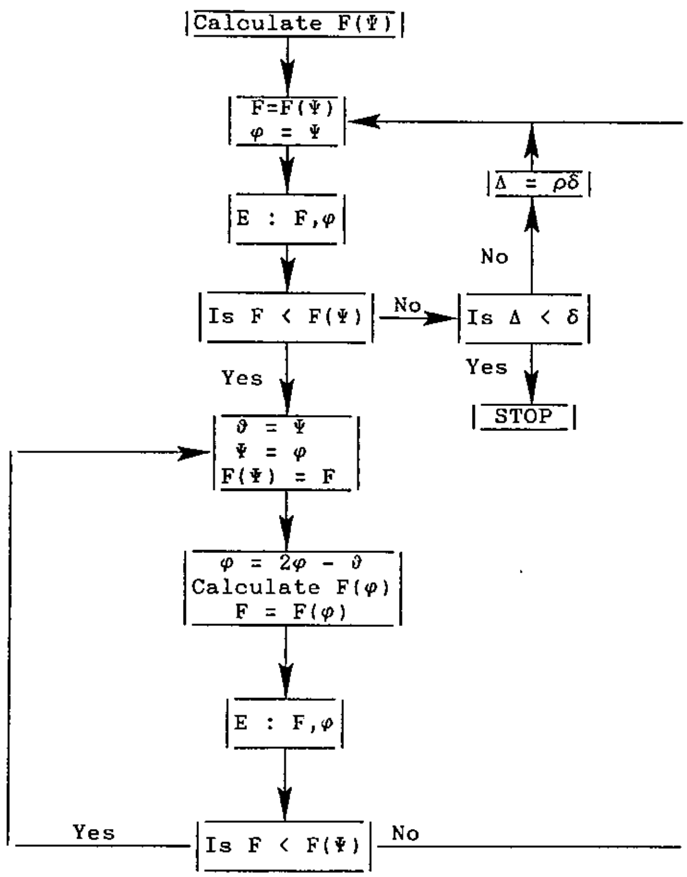
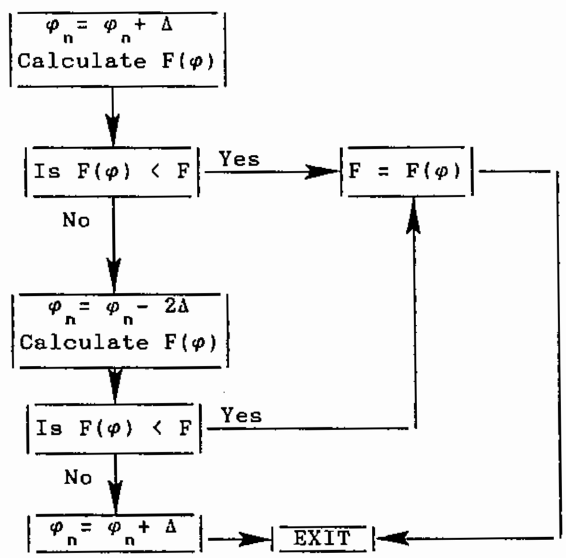


Figure 5. Flow chart for pattern move.



(This routine is carried out for each variable separately)

Figure 6. Flow chart for exploratory move.